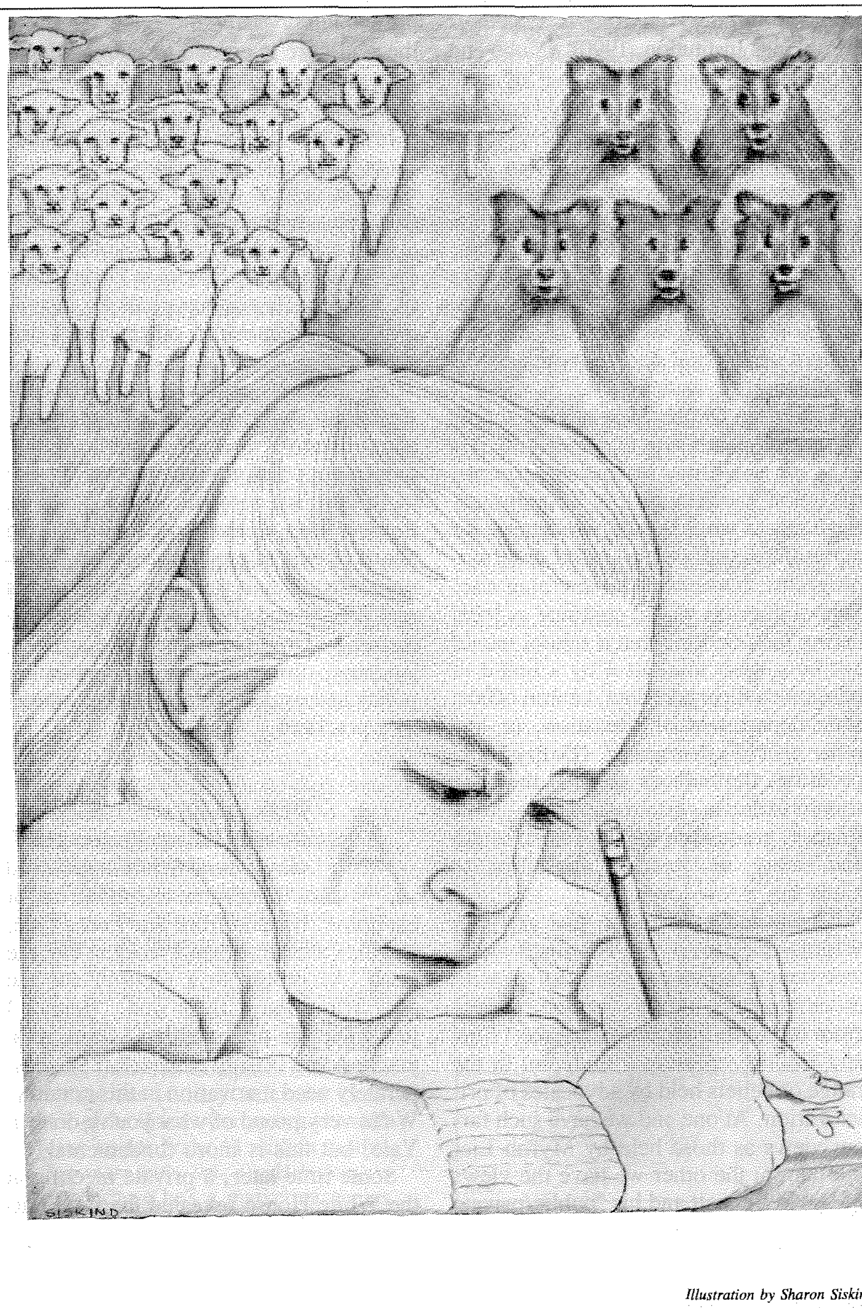
How Old Is the Shepherd?

An Essay About Mathematics Education

U.S. students go through school with serious misconceptions about mathematics. According to Ms. Merseth, parents, the popular media, and the schools themselves reinforce these mistaken notions.



BY KATHERINE K. MERSETH

EACH WEEKDAY during the school year, some 25 million children study mathematics in U.S. schools under the careful supervision of classroom teachers, using textbooks that publishers have spent millions of dollars to develop and print. With this impressive investment of human and monetary capital, why is it that our students perform as if their curriculum and instruction were on the cutting edge of mediocrity? Why is it that, on virtually every international comparison of teaching and learning of mathematics, the performance of children in the U.S. ranks near the bottom?

In order to focus discussion, consider the following nonsensical problem:

> There are 125 sheep and 5 dogs in a flock. How old is the shepherd?

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Illustration by Sharon Siskind

Researchers report that three out of four schoolchildren will produce a numerical answer to this problem.[1] A transcript of a child solving this problem aloud reveals the kind of misinformed conception of mathematics that many children hold:

> $125 + 5 = 130$ . . . this is too big, and $125 - 5 = 120$ is still too big . . . while $125 \div 5 = 25$. That works! I think the shepherd is 25 years old.

In this child's world, mathematics is seen as a set of rules – a collection of procedures, actually – that must first be memorized and then correctly applied to produce *the* answer. Examining the transcript, we see that this student is not without reasoning ability. Indeed, the accurate deduction about the appropriateness of the shepherd's age shows some sense-making. In spite of this reasoning, however, the child apparently feels compelled to produce a numerical answer. How frequently teachers see this eagerness to begin writing and manipulating numbers before the situation is fully understood!

America has produced a generation of students who engage in problem solving without regard for common sense or the context of the problem. Why is this so? Three factors must bear the blame: societal beliefs about mathematics, the typical curriculum in use in our schools, and the preparation of our teachers.

SOCIETAL BELIEFS

A number of widely held beliefs about mathematics adversely affect the ability of children and adults to experience mathematics in productive, meaningful ways. These beliefs profoundly influence the way in which mathematics is taught, studied, and understood.

Many individuals believe that mathematics is a largely rule-oriented body of knowledge that is acquired through the memorization of discrete number facts and algorithmic rules. For example, in a survey of eighth- and 12th-grade students, researchers found that 40% of students at both grade levels agreed with the statement that mathematics is a set of rules, while 50% of the eighth-graders and 25% of the 12th-graders stated that mathematics involves mostly memorizing. Lending further credence to this rule-bound conception of the field, eight out of 10 eighth-graders and two out of three 12th-graders held the opinion that there is *always a rule* to follow in solving a mathematics problem.[2] This belief in the sanctity of rules is clearly illustrated by the children's response to the shepherd problem. But in the real world of mathematics, nothing could be further from the truth.

The work of mathematicians and of those individuals who use mathematics to design such technologies as weather satellites, the Patriot missile system, or the paths that our intercontinental telephone calls follow is not governed by simple rules and formulas. Instead, mathematicians participate in a problem-solving process that is interactive and often quite fluid. Some individuals have described this process as a "zigzag path" from conjectures to explorations of the conjectures through refutations and back to reformulated conjectures.[3] Mathematicians offer hypotheses, or educated guesses, changing their ideas and their approaches in response to new arguments and discourse. When these individuals "do" mathematics, they creatively combine a variety of techniques, hunches, and ideas, constantly "re-forming" their attempts to reach a solution.[4] A rigid set of rules is anathema to the creative problem solver.

Second, many individuals believe that mathematics is a static body of knowledge. And what is taught in school reinforces this notion: most schools currently teach eight years of 18th-century "shopkeeper" arithmetic, followed by a year of 17th-century algebra and a year of geometry, basically developed in the third century B.C. Even calculus, as taught in today's schools, is three centuries old. Very few students and adults are aware that, with advances in such areas as fractals, discrete mathematics, and knot theory, more mathematics has been discovered in the last 35 years than in all previous history. Little of this new material, however, makes it into the schools or the public discourse.

This belief in the fixed nature of mathematics also influences the way the subject is presented in the classroom. The dominant mode of mathematics instruction in American classrooms is "teacher talk," with the teacher or adult "telling" the student what rule or algorithmic procedure to follow.[5] And if teachers are not talking, then students are typically engaged in silent seatwork, practicing page after page of procedures and contrived "story problems" and having little or no interaction with peers. Instead of discovering and making their own meaning of mathematics, the students' job in this environment is to memorize and absorb directions and information from others. These experiences convince elementary students that mathematics is someone else's subject – certainly not theirs. One group of researchers found that, "in math, students indicated a strong dependence on the teacher as a source of learning relative to other sources. . . . This seems to go along with the students' beliefs that they cannot learn math on their own; at least they would need some knowledgeable authority to provide assistance. Self-instruction using books or other sources is less conceivable in math."[6]

Perhaps the most crippling belief about mathematics in our society is that it is a difficult subject that can be mastered only by a very small minority – those with special gifts or abilities. A predominant view in America is that one either "has it" or one doesn't. Effort receives little credit for contributing to successful learning in mathematics – or, for that matter, in any subject. For example, American, Japanese, and Chinese mothers were asked what factors among ability, effort, task difficulty, and luck made their children successful in school. American mothers ranked *ability* the highest, while Asian mothers gave high marks to *effort*. This led the researchers to conclude that "the willingness of Japanese and Chinese children to work so hard in school may be due, in part, to the stronger belief on the part of their mothers in the value of hard work."[7]

The belief in innate ability not only minimizes personal responsibility but also fosters the view that poor performance in mathematics is socially acceptable. Many well-educated individuals proclaim without embarrassment, "I could never do mathematics!" or "I never liked the subject!" Jokes about antisocial, absent-minded, and "nerdy" mathematics professors reinforce this negative image.

These beliefs have shaped the views of many elementary teachers and are particularly damaging because they are communicated, either consciously or otherwise, to impressionable young children.

Many teachers at the elementary level feel inadequate when it comes to mathematical knowledge and ability. Consequently, in the elementary school day, mathematics takes a back seat to reading and is often relegated to the period after lunch, when students are not as alert or engaged. Mathematics and, even more so, science receive less instructional time than reading – no doubt at least partly to minimize the discomfort of the teacher.

When children come home and ask questions about mathematics, parents often answer defensively that mathematics was always hard for them or that they always found it something of a mystery. The message is, "Don't ask me about that! It's not something I value." Children's perceptions about mathematics and science are profoundly shaped by influential adults, many of whom harbor negative feelings toward those subjects.

THE TYPICAL CURRICULUM

A second factor that influences the poor performance of children in mathematics is the curriculum typically available to teachers. The mathematics curriculum in use in America is outdated, repetitious, and unrepresentative of the evolution of the field. Because curriculum is the primary means by which children learn about mathematics, this sorry situation significantly hinders achievement.

In America, textbooks determine what is taught in schools and the ways in which material is presented. Textbooks form the backbone as well as the Achilles' heel of the school experience in mathematics. The dominance of the textbook is illustrated by the finding that more than 95% of 12th-grade teachers indicated that the textbook was their most commonly used resource.[8] Reasons for the dominance of the textbook include the lack of a national curriculum and national examinations in the U.S. as well as the power of publishers and the politicized process of state textbook adoptions.

American textbooks stress computation, algorithmic procedures, and artificial "story problems." These emphases misrepresent the broad scope of mathematical knowledge. Magdalene Lampert posits that there are four types of mathematical knowledge: intuitive, concrete, computational, and principled conceptual knowledge.[9] Intuitive knowledge represents an understanding that is derived from specific contexts and relates only to those contexts. Computational knowledge enables one to perform activities with numerical symbols according to previously determined and generalizable rules. This is the most common form of mathematical knowledge presented in schools. Concrete knowledge involves knowing how to manipulate concrete objects or representations of them to solve a problem. Finally, principled conceptual knowledge represents the understanding of abstract principles and concepts that govern and define mathematical thinking and procedures. Some cognitive scientists suggest that real mathematical understanding is a combination of these different ways of knowing mathematics.

In present-day math textbooks and classrooms, intuitive and concrete knowledge receive little attention. In fact, far from being rewarded, intuitive knowledge is discouraged in some classrooms. For example, if a student solves a mathematical exercise in her "own" way, she may not receive credit because her method differs from the teacher's or text's approach, even though the result is correct.

Principled conceptual knowledge receives even less emphasis. The stress is on computation and procedure, not understanding and sense making. As a result, children come to believe, as in the instance of the shepherd, that any question in a mathematics class must have a numerical answer. In fact, the majority of elementary students in a recent study defined mathematics as dealing with numbers and the four basic arithmetic operations.[10] The current exclusion of intuitive, concrete, and conceptual knowledge from our nation's textbooks virtually guarantees that children will not be exposed to this broader conception of mathematics.

A second problem with the mathematics curriculum is an overreliance on the concept of a spiral curriculum. Stated eloquently by Jerome Bruner, the argument for such a curriculum was that children are able, at a very early age, to comprehend powerful ideas in mathematics and science. Bruner suggested that students should be exposed to such ideas early and then, as their sophistication and mental acuity increase, they should explore the same concepts in an extended way that provides for the further depth of understanding. Bruner felt that the curriculum should revisit basic ideas repeatedly.[11]

Unfortunately, the implementation of the spiral curriculum has been less than ideal. Instead of deepening a child's understanding, much of the curriculum simply rehashes the same material in the same way, time and time again. As one teacher wryly noted, "If Johnny doesn't get multiplication in third grade, he'll have another chance in fourth, fifth, sixth, seventh, and eighth grades." This repetition deadens the mind and breeds low expectations.

The percentage of new content that students meet as they move through commonly used textbooks illustrates this repetition. Between grades 2 and 8, the third grade is the only grade in which more than 50% of the material in the textbook is new to the student. And, in middle school, 60%, 65%, and 70% of the material in sixth, seventh, and eighth grades respectively is a rehash of earlier topics.[12] No wonder students find mathematics uninteresting.

Finally, textbook material is outdated and outmoded. While technological changes have transformed the marketplace and the consumer world, mathematics curricula continue to stress basic operations. Important topics such as probability and statistics (including the ability to assess one's chances on the lottery) or mathematical modeling and data analysis are either buried in the final chapters of the textbook or given no consideration at all.

As a result, children who attend our schools have few opportunities to develop the tools and the mental apparatus to understand complex situations. The Mathematical Sciences Education Board summarized the situation in American schools today:

> Of the 25 million children who study mathematics in our nation's schools every weekday those at the younger end – some 15 million of them – will enter the adult world in the period 1995-2000. The 40 classroom minutes they spend on mathematics each day are largely devoted to mastery of the computational skills which would be needed by a shopkeeper in the year 1940, skills needed by virtually no one today. Almost no time is spent on estimation, probability, interest, histo-

grams, spread sheets or real problem-solving, things which will be commonplace in most of these young people's later lives. While the 15 million of them sit there drilling away on those arithmetic or algebra exercises, their future options are bit-by-bit eroded.[13]

The possibility of a mathematically and scientifically literate citizenry will remain elusive as long as the current textbooks and the misapplied concept of the spiral curriculum endure.

THE PREPARATION OF TEACHERS

The third major cause of the disappointing performance of students in mathematics has to do with the preparation of the teaching force. If teachers prepare students poorly, it is due in large part to deficiencies in their own training. While many teachers do an excellent job, by some accounts nearly one out of every two math and science teachers does not possess adequate subject-matter training. This situation results from a fairly common practice of assigning teachers to teach classes in fields outside their areas of competence or certification. Albert Shanker has called this practice "education's dirty little secret." While the situation is pervasive, it is particularly apparent among new hires: some 12.4% of all newly hired teachers in 1985 were not certified in the fields to which they were assigned.[14] And this figure is much higher in the inner city, where the enormous challenges of the context create a perennial shortage of teachers.

Certification procedures offer little reassurance. Elementary teachers typically earn general teaching credentials for grades K-8 or K-6. Few elementary teachers take higher-level mathematics courses, and most have only one or two courses in the teaching of mathematics. This lack of training translates directly into a lack of confidence.

At the secondary level, the academic preparation of teachers appears – at least on paper – to be stronger. Secondary mathematics teachers, on average, reported taking nine semester-length courses in mathematics.[15] However, it is important to look at the content of these courses. In many cases, content was highly specialized and had little to do with material that teachers were currently teaching or would be teaching in the future. Such courses help teachers to maintain a passion for their subject and a love of learning, and certainly teachers benefit from taking them. But the issue here is one of balance. Is it more appropriate for teachers of mathematics to explore, for their own intellectual stimulation, esoteric topics that are distant from the classroom, or is it more fitting that they explore, in depth, the mathematics that they will teach in order to foster this level of understanding in children?

A second critical factor tied to the training of teachers is that teachers tend to teach the way they were taught. Therefore, it is important to look at the dominant mode of instruction in institutions that educate teachers. A vast majority of college-level mathematics classes are taught by the lecture method – a method perhaps appropriate for some portion of the college curriculum and for highly motivated learners, but decidedly inappropriate for exclusive use in K-12 schools. Clearly, there is a problem if this is the prospective teacher's only model, because it is likely that he or she will teach the same way. David Cohen and Deborah Ball, professors of education at Michigan State University, sum up the sorry state of training for mathematics teachers by asking pointedly, "How can teachers teach a mathematics they never learned, in ways they never experienced?"[16]

In addition to the challenge of devising appropriate academic training of math teachers, the teacher education community must rethink the process by which one becomes a teacher. Teachers learn about teaching in four settings: as students observing other teachers teach, as undergraduates or graduates studying in teacher education programs, as student or intern teachers in schools, and as practicing professionals on the job. Currently, these four learning environments operate in isolation from one another. A prospective teacher gains academic training in one location while issues of practice and implementation are explored elsewhere. This separation between theory and practice, content and method, produces false and limiting dichotomies that fragment and disable teacher education.

Finally, the U.S. faces a serious supply problem with respect to mathematics teachers. While some 15% to 20% of entering freshmen in four-year colleges

"I could fulfill their right to know, if they'd just exercise their right to remain silent."

intend to major in math or science, only 50% of these individuals actually graduate with a degree in either field. And the ranks continue to dwindle in graduate school, so that the number of American-born students graduating with Ph.D. degrees in math and science is an embarrassment to our education system.

To bring the problem into clearer focus, consider the following picture. Start with 100 students in the ninth grade. According to national averages, some 75% to 76% of these students will graduate from high school four years later (that percentage is considerably lower for inner-city youths). Of these 75 youngsters, 60%, or 45 young men and women, will enter four-year colleges. Of these 45 students, 40% will graduate from college four years later, leaving us with 18 graduates. Of these 18, 6% to 8% will have majored in mathematics or science. That means that, from our sample of 100 ninth-graders, one student will have pursued a degree in math or science within the "ordinary" time frame.

WHAT SHOULD BE DONE?

Given this scenario, with multiple and interrelated factors fueling the failure of many children and adults to develop a useful understanding of mathematics, what can be done? The first step is to recognize that the mathematical education of young people is entangled in a complex web of problems. Efforts with a singular focus of curricular revision, teacher education, or the retraining of experienced teachers will stretch the web in only one direction. To change curriculum without changing teaching practice or to increase societal interest while teaching the same tired curriculum would be folly. Instead, a multifaceted and comprehensive effort is necessary – one that stretches the constraining web in many different directions, causing it to break. As Lauren Resnick, a noted cognitive psychologist, says of the necessary mathematical reform effort: "We'll have to socialize [students] as much as to instruct them."[17]

Several fundamental activities are central to any mathematics reform effort and must focus on four areas: the nature of the curriculum, the nature of instruction, new forms of assessment, and beliefs about mathematics. The following discussion, while not exhaustive, outlines specific actions that, taken together, offer a plan to reverse the failure of children to gain mathematical knowledge.

CONTRIBUTIONS OF COGNITIVE SCIENCE

Both the materials and the methods used to educate children in mathematics require substantial revision. Before exploring specific suggestions, however, it is important to examine what is known about the ways in which human beings understand mathematics. These relatively new findings from the world of cognitive science should have a considerable impact on the type of materials and the approaches to instruction that are necessary to produce greater understanding.

A new consensus about the nature of learning, particularly in the fields of mathematics and science, is emerging. One of the most powerful findings in this new view is that children – and adults, for that matter – come to mathematical and scientific learning with *surprisingly extensive* theories about how the world or a particular phenomenon works. Mathematical learners are not "blank slates" or "empty vessels" waiting to be instructed. Instead, the learner, armed with personally constructed theories, creates complex schemata of understandings that influence and help interpret further learning. Cognitive scientists call these self-constructed theories "naive" theories.

In some instances, these naive theories are accurate; in other cases, they are not. What has surprised researchers is the tenacity with which people cling to these beliefs. For example, students who had successfully completed one year of college physics instruction – many of whom had received A's or B's in the formal course – were found to continue to hold inaccurate beliefs about motion.

Cognitive scientists believe that learners *construct* understandings by building relationships and connections with prior knowledge. They do not simply absorb what they are told without reflecting on it or relating it to prior beliefs and understandings.[18] To understand is to know relationships. In this paradigm, as learners learn, they store knowledge in clusters and organize these clusters into sche-

"Because daddy doesn't want *to know how to write 23 using base 2."*

mata, so that all learning depends on prior knowledge. Prior knowledge results from a combination of formal learning and naive theories. It is this linkage between new knowledge and old that explains the tenacity of "naive" theories.[19]

THE NATURE OF CURRICULUM

This new conception of the learner as a constructor of knowledge is central to any consideration of new curriculum. We need materials that evoke active, rather than passive, participation. Pages of mindless computation do not foster the construction of new knowledge. Learners need the opportunity to collect, generate, and frame their own problems and inquiries. The learner must be in the driver's seat.

In addition, topics not previously explored in traditional curricula must be added. Changes from an agrarian society to a technical/information society demand that literate citizens be familiar with such concepts as mathematical modeling, discrete mathematics, and data analysis. An example of discrete mathematics would be the decision process whereby a street sweeper is routed through a town so that the fewest number of streets possible will end up being swept twice. Many scheduling, sorting, and other processes that involve a series of "yes/no," "on/off," or "in/out" decisions make up the essence of discrete mathematics. With the exception of individual projects funded by the National Science Foundation and scattered across the country, curricula focusing on these new topics are not available.

Substantial curriculum development efforts are necessary, but they must also be undertaken in a new way. Greater attention must be given to the lessons learned from the debacle of "new math" and to our increased knowledge about the implementation of curricular ideas. New math failed because little effort was made to introduce elementary teachers to the materials and teaching approaches. Recent research on the California Framework, a newly revised state-level curriculum, tells us that it is not sufficient to introduce new curriculum in a "top-down" mode. Without substantial support, teachers simply teach new ideas in old, unproductive ways.[20] Successful curricular reform requires informed staff development and the engagement of teachers.

THE NATURE OF INSTRUCTION

The new understanding of how individuals learn mathematics and science will also have a profound impact on the methods used to teach these subjects. "Teaching as telling" can no longer be the operative form of instruction in mathematics classrooms. Instead, multiple opportunities must be provided for students to engage with mathematics.

Teachers also need to offer students avenues of exploration that will lead them to a direct confrontation with their naive theories. This means that teachers must anticipate that, as we saw with the shepherd story, children frequently rush to compute; teachers must also take time to listen to and explicitly acknowledge children's ideas. Both time and intellectual effort are essential to this process of "unpacking" children's ideas in order to foster deeper understanding. This type of teaching requires substantial subject-matter expertise as well as a willingness to share authority and control with learners.

Another important implication of cognitive science for teaching methods is that multiple representations and explanations are necessary. Individuals construct their own knowledge, and their constructions are triggered in myriad ways. While a picture may appeal to one person, a rule or an analogy may serve another. Telling students "one way" is not sufficient for the learning of all. Multiple representations and approaches are integral to successful teaching and learning.

The enhanced understanding of student and adult cognition also defines paths of reform in the education of both new and experienced teachers. Courses and workshops that stress the interrelationships between subject-matter knowledge and methods of representation and instruction must be central to both preservice and inservice training programs.

Because it will be difficult to find sufficient numbers of teachers to teach in this new, informed way, teacher educators should focus first on the education of mathematics specialists at the elementary and secondary levels. An initial goal is to place a mathematics specialist in every elementary and secondary school in the country. These individuals would have a dual role as instructional and curricular leaders in mathematics and as change agents at the school or district level. They would assist other teachers with mathematics instruction, develop and select curricular materials, and teach mathematics lessons and classes.

The training of mathematics specialists should consist of in-depth study of the subject matter, including the material taught in the schools. In addition, to better enable them to lead at the local site, these teachers should acquire a firm grounding in cognitive theory, adult development, and school reform.

NEW FORMS OF ASSESSMENT

The third focus of reform must be assessment. Changing the form, content, and objectives of assessment is the quickest way to stimulate change in curriculum and instruction. Pressure from local school boards and state legislatures for greater accountability makes testing a most powerful influence on teaching. While some may view the power of testing as dangerous, it is possible to exploit its influence in ways that will fuel rather than extinguish reform in mathematics education. Orienting the content and the forms of assessment to problem solving, critical thinking, and analytical reasoning will change classroom practice.

Whether teachers use tests provided by publishers with their textbooks or nationally normed tests such as the Iowa Tests of Basic Skills or the Metropolitan Achievement Tests, the content and style of these instruments have been profound barriers to reform. Machine-scored fill-in-the-blank or multiple-choice tests do not provide meaningful measures of higher-order thinking and reasoning skills; therefore, these skills are not taught.

Some institutions and states are moving to new forms of assessment that require students to write essays, do science experiments, and describe their reasoning processes. California, Vermont, Connecticut, and Kentucky are leading the way in this field. These new approaches are intended to test "those capacities and habits we think are essential."[21]

To reform mathematics learning, we need assessment instruments that measure students' ability to make sense of complex situations, to formulate and refine hypotheses, to work with poorly defined problems or problems with more than one solution, and to define and state problems.[22] If assessment instruments included items such as the following recent example from the Connecticut Common Core of Learning Assessment Project, educational practice would change quickly. "It is asserted that 7% of the people in the United States eat at a McDonald's restaurant every day. There are 250 million Americans and 7,000 McDonald restaurants. Is this assertion possible?" Asking students to solve this type of problem will have a significant impact on what and how mathematics is taught.

BELIEFS ABOUT MATHEMATICS

A change in beliefs about mathematics (and science) will require an unusual commitment from federal, state, and local governments as well as from the popular media. Governments can improve the image of mathematics by launching a public relations campaign to recognize and honor mathematical achievement. President Clinton could demonstrate his agility at the computer – or governors, mayors, and legislators could spend time in inner-city classrooms as teachers or aides.

In addition to symbolic support, federal and state governments can offer financial support. President Kennedy pledged to provide the human and financial resources necessary to put an American on the moon; Apollo 11 illustrated the power of such a federal commitment.

Local governments can also sponsor events that foster a positive view of mathematics. Melbourne, Australia, for example, has "mathematical trails" – walking tours of the city that highlight mathematical features of the community. Citizens are encouraged to see their neighborhoods through mathematical lenses. An activity for state and local governments in the U.S. might be to explore probability and statistics as they relate to state lotteries.

Television also holds great power for shaping beliefs. While other countries such as Britain offer television shows for adults that feature mathematical problem solving, nothing similar exists in the U.S. To produce such a program would not be difficult. Public television has made important efforts to produce children's shows that stress problem solving and mathematical and scientific knowledge. The programs "3-2-1 Contact" and "Square One TV," created by the Children's Television Workshop, explicitly appeal to children between the ages of 8 and 12 while presenting realistic problems and engaging subject matter. Unfortunately, these shows scramble for funding each season, keeping a close watch on their ratings and market share. Their existence, however, demonstrates that expertise is available to produce public and commercial television for children and adults. All that is lacking is the will.

Video rentals offer another powerful means to broaden the understanding of adults and children about mathematics. Visit a video store today, and it is nearly impossible to find a videotape that explores mathematics. With conscious effort and adequate financial support, this situation could change, and videos could be produced that teach as well as entertain.

Newspapers, magazines, and radio also influence mathematical understanding. While some newspapers devote a particular day of the week to "science news," much more could be done. Why not a section in every issue? Why not articles written for various age groups on specific topics? And what about other media? Milk cartons, candy bar wrappers, and cereal boxes are held and examined by millions of children each day. What would it take to print a "math message" on each? Why not introduce "math minutes" on radio stations, to be broadcast prior to each sports report or stock market summary?

Some activities, such as television or video productions, are costly, while others, such as walking tours and milk carton messages, are not. Does America have adequate financial resources to undertake these activities? The answer must be yes, if there is a conviction that mathematics can improve the quality of life for all citizens. The popularization of mathematics does not have to remain a fantasy.

Changes in the mathematics education of our young people will depend on many individuals working in multiple areas and sharing a common vision of what is possible. It is within our reach to have highly literate citizens who will read the story of the shepherd and smile, knowing that the data are insufficient to determine the age of the shepherd. Mathematics need not be the purview of the few; it must be made available to everyone. That is a goal our society can and must achieve.

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