An Inquiry into Inquiry-Based Learning in the Mathematics Classroom

My Background/Motivation for Research

For quite some time now, I have had a desire to improve my effectiveness as a teacher of mathematics at the high school level. During my career, I have succeeded in meeting the demands of the administrations under which I have served by increasing the results of my students’ standardized tests, but I have always felt that I should and could do more to engage my students and promote curiosity. For a very long time I have been interested what really good teachers do in the classroom and my research led me to the Masters in Mathematics for Educators here at Harvard University. I have learned by example from some of the best professors of math and I emulate many parts of each of their styles. I have become interested in the ideas and potential benefits of teaching a more student-centered/discovery class. Project-Based or Inquiry-Based Learning is well-defined by Vicki-Lynn Holmes and Yooyeun Hwang, professors of education at Hope College, as “students working collaboratively to design solutions for authentic and meaningful questions and problems in the real world” (Holmes and Hwang, 2016). IBL and PBL advocates maintain that students who study in this way are more motivated. In mathematics, Holmes and Hwang report that “research indicates that PBL , with its unique hands-on (interactive and collaborative), minds-on (authentic problem solving and critical thinking), approach to learning, increases deeper understanding of mathematical concepts; helps students retain knowledge learned and enhances students’ ability to apply knowledge in real-works scenarios” (Holmes and Hwang, 2016). Intuitively, the idea that the student should discover his or her own information with minimal guidance from the instructor, is compelling and deserves further evaluation.
When I was in fourth grade I remember sitting in the front of the class in double tables with my good friend Bonnie Brown working with the small brown “math paper” on problems that made sense to me and that I could do with a confidence that I did not have in other subjects (or for that matter socially). Just before the recess bell, Bonnie and I were told by the teacher that “two heads are better than one” and that we should be doing our work individually. In essence, we were told we were cheating for having done so much collaboration. I felt ashamed. Ironically, today, as a teacher of mathematics, I struggle to find ways to encourage collaboration and engagement and to empower my students to feel that delicate and fleeting confidence that was taken from me back in fourth grade. I am not alone in this pursuit and I have found that the discussions abound and the answers are as complex and varied as the number of students in my class each year. The alphabet soup that has been created by the great number of studies and conclusions is worth examining. The real answer may be as idiosyncratic as the learners themselves.

**The Need for All of us to become more effective teachers.**

There is definitely a need to improve student performance in mathematics. Business leaders complain that new hires in STEM professions “tend to have rote calculation ability but are unable to think critically and use structured problem solving techniques to apply mathematical concepts” (Donnelly, et al, 2017). Also, these business leaders maintain, their new hires struggle with collaborative work and with communicating their ideas and technical answers clearly (Donnelly, et al, 2017). At the same time, back in the classroom, our students are asking, “Why do I need to know this?” Clearly there is a link in the educational chain that has broken or is missing.
Constructivist, IBL, PBL, LRI

George Polya of Stanford University wrote an article in 1963 that for many is the gold standard of mathematics teachers. He wrote of his philosophy of teaching and, he said, it matters little that other teachers take his point of view, but what matters is that teachers have a philosophy of teaching at all. One of his main goals in teaching, he said, was to ‘Teach to think’ which means that “mathematics teacher should not merely impart information, but should try also to develop the ability of the students to use the information imparted: he should stress know-how, useful attitudes, desirable habits of mind” (Polya, 1963). Mr. Polya could be described as a practicing IBL or PBL teacher. He said “mathematical thinking is not purely ‘formal’; it is not concerned only with axioms, definitions, and strict proofs, but many other things belong to it:...Let us teach proving by all means, but let us also teach guessing” (Polya, 1963). Polya’s essay is very persuasive but not supported with data.

I have found that there is an abundance of “experts” naming an abundance of better ways of instruction (e.g. Design-based learning, Game-based learning, Inquiry-based learning, passion-based learning, Inquiry-based learning, Proficiency-based learning, Team-based learning, etc.). Each claiming that the students will retain more, learn more, be more able to think critically and adapt concepts, communicate and collaborate better and generally do better in their future life. The authors create new acronyms and appear to recreate with various twists, Jerome Bruner’s original idea of 1966 (Bruner, 1966). My research has suggested that an attempt on my part to define and locate the origins of each of these forms of discovery based learning is entirely up for interpretation. I am not sure that one led to another in a sequential development. I believe that experts from the fields of psychology and education and science and math are all noticing the need and somewhat independently, coming to the rescue. I will attempt to define discovery-based learning, its possible origin, and its purported solution in this paper.
The overarching idea behind the unguided approach to teaching is that the students discover the concepts for themselves and thereby develop a deeper working knowledge of mathematics. There are many math teachers like myself interested in exactly what is involved in creating a discovery-based lesson and classroom. The list of techniques that are considered “discovery-based” is extensive and includes such activities as “interactive lectures, collaborative groups, student presentations, board work, computer lab activities, games, and challenging homework problems” (Hodge, 2006). My research revealed several accounts of teachers (Hodges, 2006; Wernet, 2016; Professor Judah Schwarz of Harvard University) who have made the intentional switch to a more discovery based approach and their comments on the changes are similar. These educators uniformly say the change is hard and is a lot of work and takes time. They say a lecture can be discovery based so long as the right provocative questions are asked. Some mention that small attainable goals are the best and that the teacher’s personality does not have to be a factor in the difficulty of the change (Hodges, 2006). The complaint that less is covered is not frivolous and plays into the decision by the teacher to weigh the amount of discovery based teacher he or she will do. It takes time to allow the students to discover and very often this is valuable class time that might be used to cover necessary curriculum-dictated subject matter.

The idea that a class in mathematics can be taught by giving the students a small amount of background information and a problem to which they must independently discover a solution is often attributed to a Mr. William Kilpatrick who is known worldwide as “Mr. Project Method” (Knoll, 2012). Mr. Kilpatrick published a paper in 1918 on the subject and his celebrated use of a student-centered collaborative approach to teaching. But, it turns out, this method of teaching can actually be traced back to 17th century Italy and can be found to have been used in the US as early as 1865 in Agricultural Education and General Science (Knoll, 2012).
The instructional practices encouraged and outlined by the National Council of Teachers of Mathematics advocates mathematics courses where students can “learn mathematics through exploring and solving contextual and mathematical problems” (Principals to Actions: Ensuring Mathematical Success for All – NCTM). The goal is to create persistent problem solvers who communicate and collaborate with peers and feel confident in their understanding. What the NCTM appears to be encouraging is little different from what Mr. Kilpatrick promoted in 1928.

Inquiry-Based Learning is a step beyond Problem-Based Learning in its unguided approach. IBL is commonly attributed to Professor Robert Lee Moore who taught mathematics at the University of Texas from 1950 to 1969. Moore would hand select his mostly white, male class so that it was “as homogenously ignorant as possible” (Haberler & Laursen, 2018). He would then provide his students a few axioms and definitions and leave them completely on their own to prove a set number of theorems. He would expect them to come to the next class with their proofs completed on their own without use of a textbook and with minimal definitions and symbols. He would then call on his students from the weakest to the strongest to begin demonstrating proofs of the theorems. This “Moore Method” was well regarded by some at the college level because of its student-centered and inquiry-based nature. Students were “doing rather than seeing, hearing, or reading” (Pradaig, 2009). However, there are two major distractive issues with this method of teaching/learning. First, Professor Moore was known as a racist and sexist and, because of this, his method of teaching did not sit well and could not be separated from his poor reputation (Haberler & Laursen, 2018). Additionally, the” Moore Method” requires more time for the students to come up with the theorems on their own. As a consequence, less material is covered. Moore and others would argue that the material covered is more well understood, but it is a much less efficient means of learning. Students of Professor Moore remembered his classes positively and those that went into the education profession began using this “Moore Method.” As a result, this
very early form of IBL became a grass roots movement at the college level, not a dictate by the state or federal education pundits. Inquiry-Based Learning of the Moore variety has been studied for its short term and long term outcomes for student groups at the undergraduate level. These studies produced positive outcomes of “deepened understanding, confidence in their mastery of mathematics, heightened communication and collaboration skills, and appreciation for mathematics as a way of knowing. Students feel empowered by IBL experience, especially women, and students’ grades in later courses suggest lasting benefits...” (Haberler & Laursen, 2018). However, I have found no example of the “Moore Method” used in high schools and, consequently, very little study on the effects of IBL at the high school or middle school level.

Today, there are many college level institutions that practice the “Moore-method” under the guise of Inquiry-based learning and some use a modified Moore Method (“MMM”) which moves at a slightly faster pace: “...not too fast (as perhaps in a traditional German seminar – recitation- method) nor too slow (as perhaps in a constructivist or pure Moore method course)” (Padraig, 2009). Some maintain that true learning and understanding can only really happen when the student is thinking and completely invested in the subject. “No thought, no understanding. So, mathematics education should be centred on encouraging a student to think for one’s self: to conjecture, to analyse, to argue, to critique, to prove or disprove, and to know when problem is solved correctly, to know when argument is valid or invalid.” (Padraig, 2009).

I have been able to find plenty of discourse at the college level about the merit of Instruction-Based Learning. One very large study from college mathematics involved 100 sections of 40 courses and 3,212 students. The goal of the study was to evaluate several issues surrounding IBL courses including the issues of grade inflation, lack of rigor and slower pace of the courses. The results, according to the author, Sandra L. Laursen was that the overall effect of “IBL on students’ subsequent grades and course-
taking was modest when comparing IBL and non-IBL students in their entirety. Certainly no harm was done,” (Kogan and Laursen, 2014). There are fewer studies on the subject at the high school or middle school level in mathematics. There is, however, some discussion around the merits of a modified Inquiry-Based style of teaching in the science classroom. The use of the POGIL (Process Oriented Guided Inquiry Learning) in the chemistry class at my school (The Bromfield School) is an example of this sort of guided inquiry-based teaching at the high school level. The science teacher produces a packet of guided questions and the students work collaboratively to answer them. In another interesting study completed in 2017, a class of eighteen high school honors calculus students participated in PBL lessons involving real situations. In one problem, for example, a lawyer defending a client sought assistance in proving that his client could not have caused permanent harm to the pilot of an airplane by pointing a laser pointer at him as he was flying. The students had to figure out what level of exposure the pilot experienced at different target distances. Students had to make assumptions about the plane's speed, the wind conditions, the effects of windscreen glass and other true-to-life questions (Donnelly, et al, 2017). The pre- and post-tests completed by this group of students were the same used widely to measure college students' motivation and strategies for learning. The test is called the Motivated Strategies for Learning Questionnaire (MSLQ) (Donnelly, et al, 2017) and the results from the high school students revealed that there was “no significant change in student motivation between pre and post MSLQ surveys except for a large positive increase in response… to the statement “I’m certain I can master the skills being taught in my classes.” And there was a significant decline in the response to the statement “It is important for me to learn the course material in my courses” (Donnelly, et al, 2017). This study concluded rightly that “similar studies could be done on a wider range of students, both in terms age (sic) and topic study…”. I agree and remain frustrated that no such study has been done to
demonstrate without question that a student-centered, inquiry-based lesson or classroom is more effective than that of the direct instruction.

Today, there is an increasing focus on experimentation and inquiry skills in educational large-scale assessments like the “Programme for International Student Assessment” (PISA), and the “Trends in International Mathematics and Science Study” (TIMSS). Despite these grand international studies and developments, an important question remains open and unanswered: “Does inquiry-based teaching result in higher ... achievement?” (Teig, 2018). Again, the research on this question is sparse and conflicting. In one research study of the science classroom, the researchers found that, in fact, the relationship between student achievement and inquiry-based learning is not linear. The time spent on inquiry activities is time that could be spent on practicing and necessary teaching. Thus, as the inquiry-based activities increased, the resultant learning and achievement decreased. The authors of this study (Teig, 2018) suggest that the relationship between increased inquiry activities and learning is “curvilinear” rather than linear (Teig, 2018). That is, “increasing use of inquiry-based teaching was correlated with higher achievement in science until it reached an optimum value, then this association decreased as the use of the strategy increased” (Teig, 2018).

The Other Side of the Discussion

There are those who argue vehemently that regardless of the nomenclature (i.e. problem-based, Inquiry-based, experiential, constructivist), the idea that teaching can be an activity where you allow the students to discover facts, theorems, answers to problems on their own is ineffective and results in negative outcomes. Further, these pundits maintain, these ideas are defended on the basis of intuition alone (Kirschner, 2018). In 1985 research entitled “Worked Example Studies” proved that novices studying examples that had been previously worked out for them in class, learned their subject
much faster and better than those who were tasked with working out the solution themselves (Kirschner, 2018). The explanation for this lack of correlation between inquiry-based teaching and actual learning is that our brain’s working memory is only capable of holding a certain number of elements at a time. If we use up our working memory in the search for a solution, then there is no room for the actual solution which then can be transferred for permanent storage in the long term memory. Learning is the act of storing information in the long term memory and the interaction between the long term memory and the working memory is key to the process. There is a limit to the amount of new information the brain can process at one time (Center for Educational Statistics and Evaluation, 2017). Earlier pundits like Jerome Bruner who promoted student-centered learning were not as well aware as we are today of the workings of the brain (Kirschner, 2018). There are some who believe that a less structured approach might be useful for those who are “further along the novice/expert continuum if such instruction is designed with the constraints of working memory in mind” (Center for Educational Statistics and Evaluation, 2017). Cognitive Load theorists point to a large number of random controlled trials (RTC’s) that support that contention that direct instruction is more useful when teaching novice learners in mathematics and sciences. Less is known about the effectiveness of this sort of teaching in less technical subjects like literature and history (Center for Educational Statistics and Evaluation, 2017).

According to Paul A. Kirschner (Kirschner, 2018), “(e)ach new set of advocates for unguided approaches seemed either unaware of or uninterested in previous evidence that unguided approaches had not been validated. This pattern produced discovery learning, which gave way to experiential learning, which gave way to problem-based and inquiry learning, which now gives way to constructivist instructional techniques.” The well-regarded educational psychologist, Richard Mayer of the University of California whose studies focus on how people learn, said that the “debate about discovery has been
replayed many times in education but each time, the evidence has favored a guided approach to learning” (Mayer, 2004).

**Am I Asking the Right Questions?**

To this point, my research concludes that many (it seems even most) educators today intuitively believe that a discover/inquiry-based lesson will promote learning and understanding to a far greater degree than the methods of direct instruction. And yet I found little data to support this belief at the secondary school level. The advocates of direct instruction stand by their understanding of how the brain functions. If we are measuring the amount of learning achieved by our students, these educators report, the data proves direct instruction is the most effective method. Both sides have compelling arguments and I begin to wonder if, perhaps, I am asking the wrong questions. Do I want to be an effective teacher who produces students who have learned what I am conveying?; or do I want to foster curiosity and interest that these students will have for future lessons and for life? If I want to achieve the latter, then maybe I do not necessarily need to adopt IBL, or PBL, or Moore, or any specific learning method. Maybe I simply need to foster an environment that promotes curiosity and engagement and interest. A three-year study by Katie Makar and Jill Fielding-Wells concluded that the classroom environment may be a large factor in fostering such an environment. This study by Makar and Wells looked at “classroom norms” (Makar and Wells, 2018) and tried to find commonalities in classrooms that adopted norms that “promote curiosity, risk-taking and negotiation needed to productively engage with complex problems” (Makar and Wells, 2018). In every classroom, this study reports, there is a set of norms both explicit and implicit and “through these norms, children develop a view about the discipline of mathematics and what is valued and important in that discipline” (Makar and wells, 2018). Makar and Wells provide a useful example of the two kinds of norms established by the teachers in two different classrooms in the following interaction:
This interaction demonstrates how simply the correct provocative questions and responses can establish a classroom that is collaborative, curious, and inquiry-based. This study included a database of over 1000 videotaped lessons from 54 teachers. The study concludes that to “introduce an approach like mathematical inquiry with significantly different goals and philosophies is unlikely to succeed without changes to classroom social and sociomathematical norms...” (Maker and Wells, 2018).

**Concluding Thoughts**

George Polya says that it “is your duty as a teacher, as a salesman of knowledge, to convince the student that mathematics is interesting, that the point just under discussion is interesting, that the problem he is supposed to do deserves his effort...If we wish to stimulate the student to a genuine effort, we must give him some reason to suspect that his task deserves his effort” (Polya, 1963). The question remains: But how? More research needs to be done on the effects of Inquiry-Based Learning in high school mathematics classes. These sort of large and, by many accounts, difficult changes to the classroom should not be insisted upon by the pedagogical pundits on the basis of intuition alone. Paul
Kirschner maintains that learning means to alter long term memory and, as teachers, we must give learners specific guidance about how to cognitively manipulate information in ways that facilitate storage in the long term memory (Kirschner, 2018). If, as teachers, we think this is our main goal, then there are plenty of studies which demonstrate that direct instruction is the more effective process for teaching. If, as teachers, we think our main goal is larger than ensuring our students have the right answer on a standardized test. If we believe, as Polya does, that we should be creating life-long learners who are sincerely interested and curious and who can think and problem solve in the real world, then perhaps we need to pursue and create more inquiry-based lessons and classroom norms which, by most studies that I have found, appear to, at the least, do no harm and, at the best, create such learners.

Matthew J. Donnelly, et al created the following “PBL Implementation Continuum” for their study (Donnelly, et al, 2017):

![PBL Implementation Continuum](image)

The diagram is a good guide for teachers like me struggling to improve. As teachers we can intentionally place ourselves somewhere on this continuum. At the outset of my research I wondered how I can become a more effective teacher. I have found that the answers are as idiosyncratic as the students. Until further studies are complete, teachers must make personal judgements.
References


Knoll, Michael. “‘I Had Made a Mistake’: William H. Kilpatrick and the Project Method.” *Teachers College Record* 114, no. 2 (2012).


