# The Benefits of Review in the Math Classroom

Math Education Seminar Series

Capstone Paper

**Introduction: Why I Chose to Research Cumulative Review**

Before delving into the topic of cumulative review, I want to give context into why I chose this topic. As a math teacher, I have often found that many students have a difficult time retaining material from the beginning of the year. As a math teacher, this becomes a frustrating situation when students need to take cumulative tests, like midterms and finals. When they start reviewing, it is clear that students often need to be "retaught" the material and are unable to "recall" the material from the beginning of the year. I have also found that almost everything that they have already learned in past years looks familiar to them, yet they have a difficult time knowing what tools or how to actually solve problems that they have supposedly already "mastered." This also becomes frustrating as a teacher when I need to "reteach" concepts that students claim they do not remember or were never taught.

Many questions arise from these frustrations. Why is it that students seem to have mastered a concept but are unable to reproduce it after some time has passed? If students need to see an example of an old problem done out again to remember how to work out a problem, have they truly mastered the concept or have we only trained students to work robotically and not necessarily have the depth of understanding that they should? Do we need to pull out problems from past units every now and then for students to attempt again so that they do not forget "how to do" a problem? Or should we be incorporating previously acquired skills into the new concepts that they are currently learning in the classroom? Is there too much compartmentalizing in the curriculum with respect to concepts and problems throughout the year and should we be linking concepts and units within the curriculum more clearly with students?

As I ponder these questions regularly as a teacher, I decided to look into any research available regarding cumulative review and mathematical retention, in the hopes of finding applications for my own teaching practice. One of the things I found in my research is that cumulative review can really only be understood and implemented after understanding the types of practice that are given to students. Otherwise, cumulative review can look different from class to class. Thus, I started my research by first defining the different types of practice.

**Different Types of Review and Practice**

In the classroom, cumulative review is generally understood to be a review of all the concepts that have been taught up to the present time in a class. For example, in a math classroom, a cumulative review for the midterm would consist of all the topics that have been taught since the beginning of the year up to the most recent lesson. However, it is difficult to label something as “cumulative review” without first understanding the different types of practice that are given.

There is a general consensus on categorizing the different types of review or practice that are done in the classroom, although they are referred to with different names. These are typically called blocked practice, mixed practice, and spaced practice.

Blocked practice, as defined by Rohrer (2009), is “a group of consecutive problems… devoted to the immediately preceding lesson.” This type of practice is also known as “massed practice.” This kind of practice gives students problems that refer only to the lesson that students were last taught and does not necessarily use any of the skills that students may have learned earlier in the chapter, unit, or term. Textbooks often give practice in this manner, and typical problems that are given focus on reinforcing a particular skill and typically do not require a high amount of mental effort once a student knows the steps that are required to solve the problem.

Blocked practice is a type of review that allows students to review the content that they have just learned and reinforce these new concepts. Students know which concepts will apply to the problem before they even read the problem. Massed practice like this helps students to internalize a specific process to the point where they do not need to think about the concepts related to the problem, and they can become very fast at applying a specific strategy or skill. Blocked practice encourages students to identify and recognize the similarities between problems that can be solved in similar ways (Carvalho & Goldstone, 2014). While blocked practice is technically a type of review, as students are *reviewing* concepts that they learned in the most recent lesson, the term “review” most often refers to one of the other two types of practice.

The next type of practice is mixed practice. Also known as interleaving, mixed practice is understood to be a set of practice problems that draw from concepts from multiple lessons and are presented in such a way that the problems are not ordered by topics or side-by-side with similar problems. While blocked practice relates completely to the most recent lesson, mixed practice can be drawn from a variety of content over a wider time range. A set of mixed practice problems could be based on the last month of material, the last 3 months of material, or even material taught over an entire year. The key feature of mixed practice is that the problems are presented in an order such that the student does not have a particular context paired with a problem.

Mixed practice not only gives students an opportunity to remember what they have learned in the past, but it also “gives students the opportunity to recognize which features of a problems are relevant to the choice of concept or procedure” (Rohrer, 2009, p. 10). Aside from mathematical skills, the ability to determine what tools, skill sets, or concepts apply to a problem, is in itself a skill that mixed review pushes students towards that blocked practice does not.

The third type of practice is spaced practice, also known as distributed practice. As its name suggests, spaced practice is understood to be practice problems that are given with spaces of time in between each set of problems. Spaced practice allows "an individual [when] confronted with a repetition… [to make] an attempt to remember, that is, to 'retrieve' or 'access' the previous experience with the repeated information" (Dempster, 1991, p.73).

This seems similar to mixed practice. However, it does not necessarily contain a mixture of problems, but rather, it can be a type of problem, given at different time intervals (one week later, one month later, etc.). If the space of the time interval is shorter between practices, the students' memories of their previous experience with that type of problem will make it easier to retrieve that mathematical concept or skill. With a lengthier space of time between practices, it will become more difficult for a student to retrieve this memory of a previous encounter with the material and will force the student to use more effort and concentration in order to obtain the skill from memory.

While there are multiple types of practice in the research literature, cumulative review can refer to mixed practice, spaced practice, or a combination of both. As Carpenter (2014) indicates, mixed practice problems naturally require problems to come from different sections or lessons, so mixed and spaced practice are often connected with each other, and we will consider both as being characteristic of cumulative review. Kang (2011) also claims that it is not only the spaced practice that helps students, but because the spaced practice naturally lends itself to mixed practice, mixed practice is also a key component of cumulative review that is beneficial to students. With this in mind, we will look into why cumulative review should be implemented in the classroom.

**Why Implement Cumulative Review?**

There are a myriad number of reasons that researchers have found for implementing cumulative review in the classroom. The only reasons for *not* implementing cumulative review are related to time constraints or the amount of material in the curriculum that remains to be covered, and not due to the nature of cumulative review itself. However, before allowing these reasons to keep teachers from implementing cumulative review, we will take a look at all the reasons *for* implementing cumulative review.

The first, and probably most obvious reason, is to help students retain information and skills that they have learned. As Suydam says, “Review helps children synthesize what they’ve learned and helps them to identify what they haven’t learned” (1985, p.26). Cumulative review helps students practice going back to everything that they have learned and recall an old skill that they have previously acquired to solve the problem, or as Rohrer (2009) coins it, “retrieval practice.” Review also allows students to determine what they remember and what they do not remember and properly go over the material that they have forgotten, a strategy that helps students in studying, especially when it comes to reviewing for a test and determining what concept or strategy they need to go back to and relearn or review. According to Carvalho & Goldstone (2014), the process of cumulative review, especially in the aspect of spaced practice, helps students in keeping what they have learned in their long-term memory. Spacing has been found to benefit learning after a student’s first exposure to the material, and the longer the space of time since their first exposure, the better the student’s final retention will be (Cepeda et al., 2008).

Another reason for cumulative review is that it gives students a deeper understanding of the material. Blocked practice does not allow students to engage with the content as deeply.

However needed it is when a new idea or concept is introduced, students will likely only have a shallow understanding of the new concept until they revisit it in cumulative review. Cumulative review allows students to engage with the material in a new way, where they are required to more actively process, recognize, and recall the skills needed to approach the problem (Dempster, 1991). By doing so, cumulative review forces students to look for key words and link them to concepts they have already learned, creating a stronger understanding of previous material. Thus, cumulative review is necessary in order for students to have a deeper understanding of the content.

Another reason for cumulative review is to give students the opportunity to practice determining what mathematical concepts or skills are required to solve a problem. This particular skill of being given a problem without a specific context forces students to utilize a different part of their brain that is not used when they simply apply a mathematical procedure or concept to a problem when they are given a specific context. As Rohrer states,

> "Discrimination ability is difficult to acquire in mathematics, in part because superficially similar problems often require different strategies… the difficulty of identifying the appropriate concept or procedure is not limited to procedural problems…. The difficulty of a word problem is typically due in large part to the absence of an explicit reference to the appropriate concept or procedure" (2009, p.10).

In addition to utilizing this part of the brain, Mayfield and Chase found that "incorporating cumulative practice into training procedures will lead to high levels of performance on novel, untrained skills" (2002, p.119). When students practice this skill of identifying the underlying concepts in a problem, they are practicing the process of understanding a problem that they have never seen before and determining the underlying skills and concepts that they may need to solve the problem, perhaps even outside of the classroom. As Rohrer puts it, "Proficiency in mathematics is measured solely by the ability to solve problems, and this in turn demands that students learn to distinguish between superficially similar kinds of problems requiring different strategies" (2012, p. 2) Students must not only be able to apply a skill when given the context, but being a literate mathematics student means being able to determine what skill or concepts to apply in a problem without the given context. As Kang (2011) found, this "discriminative contrast" aids in helping students have a better understanding of the material, in being forced to determine the tools they already have to solve a novel problem. This skill of approaching problem-solving using this discrimination ability is in itself an important life skill that students will need to have, whether or not they enter a STEM-related field, and it is a skill that can and should be trained in the mathematics classroom.

Another reason for review is to help develop and bolster students' positive attitudes towards mathematics. While it might be difficult to see how reviewing old concepts would help to do this, if review is done consistently in the classroom, Suydam found that "[review] can help [students] develop confidence in their ability to be successful in mathematics" (1985, p.26). Helping students practice using their discrimination ability allows them to become better at approaching an unfamiliar problem. When a student becomes better at approaching an unfamiliar problem, they are then more confident in their abilities.

Rohrer describes the way in which cumulative review can be used to change a student's attitude towards mathematics: "the variety of problems within each practise set that inherently arises with a distributed practise format might reduce the monotony of the assignment while also presenting an additional challenge" (2006, p.1219). He adds that this added challenge can be seen by students as more worthwhile to them, providing students with an instance of what he calls "desirable difficulty." Giving students an opportunity to do problems that are not all similar to each other breaks free from the monotony that blocked practice often falls into and the additional challenge, when framed in an appropriate manner, can be used as a point of motivation for students.

Dempster (1991), also states that properly spaced reviews helps to improve classroom learning and retention, which helps to sustain positive classroom attitudes. When students are able to retain more of what they have learned, including concepts that they have learned in past years, they are able to pull from their previous knowledge to aid in providing context to new ideas and skills. With this context from retention, students are able to sustain a more positive classroom attitude when they have the skills and tools readily available for them to apply. A positive attitude towards mathematics also helps when students are faced with a more difficult problem, while a negative attitude towards mathematics will more than likely cause a student to shut down when they are given a harder problem.

These reasons are sufficient for teachers to consider prioritizing cumulative review at a much higher level in the classroom than is typically the case. Research in this field of cumulative review in the classroom supports these reasons, providing evidence that the implementation of cumulative reviews has these positive effects.

**The Effects of Cumulative Review**

One benefit of cumulative review that Rohrer (2006) mentions is that students who do not understand the current lesson have the opportunity to work on practice problems from previous lessons that they do understand as well as work on problems related to the current lesson at a later time. Revisiting topics gives students the opportunity to be exposed to the material again and have the opportunity to understand it when they see it a second time. Since practice problems on the topic will be given in cumulative review, students will also have the opportunity to practice problems on the topic at a later time, even if they do not have a complete understanding the first time around when blocked practice was likely given.

In addition to the logistical aspect of cumulative review, research has found that reviews not only allow students to revisit material that they have already learned, but they actually help students understand the material on a deeper level. Dempster states that "reviews… do more than simply increase the amount learned; they may shift the learner's attention away from the verbatim details of the material being studied to its deeper conceptual structure" (1991, p. 71). When students understand how to apply a mathematical concept to a problem outside of the context of the lesson in which it was taught, students gain an understanding that they had not previously had when first learning the content. Both spaced repetition and mixed practice contribute to this new depth of understanding.

Spaced repetition promotes a thought process that is much more constructive (Dempster, 1991). This type of thought processing requires students to become more actively engaged with the material, as opposed to employing a rote process to solve a problem. They consciously interact with the material to a greater extent and are forced to think on a higher level in cumulative review, whereas students can work through a set of blocked practice problems with much less thought. The spaced practice aspect of cumulative review helps to foster mental processes like concentration and focus that are vital to a math student's educational development.

Mixed practice, or interleaving, also contributes to this depth of learning. When students study different concepts and alternate between the different concepts, they are actually differentiating among the different types of problems, as they are practicing using their discrimination abilities. (Carvalho & Goldstone, 2014). When students try to retrieve a concept from memory, they compare a novel problem they are confronting to previous problems that they have seen before, experienced, and solved, stimulating their minds to recall the process for these previous problems as well.

Multiple opportunities to review and revisit material so as to increase understanding the material on a deeper level are not the only effects of cumulative review. A more complete and deeper understanding also lends itself to higher test scores and a higher competency in learning new material. Rohrer found that “test scores… dramatically improved by the introduction of spaced practice or mixed practice… both features increase efficiency as well as effectiveness” (2009, p.15). In addition to tests in the classroom, Mayfield and Chase (2002) found that cumulative practice has had a positive effect on tests such as college boards. In addition to higher test scores, research has also shown that cumulative practice makes students better at and more efficient in acquiring new skills. Lee found that “review questions, when provided with feedback, enhance the retention of transfer skills in mathematics” (1980, p.334). Cumulative practice requires students to develop the skills that lead to a higher competency and proficiency in approaching a new problem.

With all these positive effects of cumulative review, we must figure out the best way of bringing this into the classroom more regularly. With all the research that already exists in favor of cumulative review, which consists of both mixed practice for training students’ discrimination abilities and spaced practice for training students’ retrieval abilities, we need to implement cumulative review in the classroom and go beyond students’ abilities to quickly learn a process and apply it to an assessment before they forget the information or process.

**Application of Cumulative Review in the Classroom**

In doing my research particularly on cumulative review, I found that the application of review in the classroom is multi-faceted. Each type of review or practice enhances a particular part of the classroom experience for the student.

Blocked practice is necessary to classroom learning. As Rohrer says, “After a certain amount of practise is completed, additional practise is beneficial only if the initial practise has undergone consolidation, a process by which memory traces re-strengthen during the time period immediately after the learning episode” (2006, p. 1218). Although most research shows that cumulative review is much more effective for a more holistic understanding of the material, blocked practice is the first exposure students have to solving a problem of a certain type when they are first introduced to a concept and need to understand the process of solving this type of problem. Most of the research on review explores cumulative review, as blocked practice is typically already a part of the mathematics classroom. Most textbooks are also already arranged by topics or ordered in a convenient and easy way for teachers to find problems by topic or type, lending the textbooks themselves to being easily used for blocked practice for homework or classwork.

Most of the research included some suggestions for application of cumulative review in the classroom, so while blocked practice may be necessary, the research provided key suggestions for cumulative review, both as mixed, or interleaved, practice and spaced practice. The following highlights the most useful application points.

Reviews given right after a student is initially exposed to a concept are useful and needed. However, Rohrer finds that “the first one or two practise problems might yield a large increase in a subsequent test score, but each additional practise problem would provide little or no gain unless it is delayed until a later session” (2006, p. 1218). As mentioned before, practice problems relating to the content just taught is absolutely necessary. However, having students complete a “mass” of problems relating to the content has little effect on the depth of their understanding. This is perhaps where blocked practice, or massed practice, acquired its name. A massive number of practice problems has a small effect on a student’s understanding, although a couple of practice problems is necessary to give students an opportunity to work through their understanding of a new concept.

Reviews must be incorporated in a systematic way into the curriculum. Students must see that reviews are a regular part of the curriculum and should not be surprised to see problems that they have learned in a previous chapter after a period of time. Carpenter (2014) suggests including at least brief mixed practice as part of homework or classwork assignments. Suydam (1985) suggests reviewing at different time intervals: on a daily, weekly, and monthly basis. Daily review allows students to cement new concepts and solidify a new process or pattern of solving a problem. Weekly, monthly, or even further-spaced review allows students to relearn any material they may have forgotten. Reviewing at these different time intervals is a way of creating spaced practice for students, of which we have already discussed the benefits. Also, by reviewing at different time intervals, students have a better idea and sense of the overall mathematical ideas or direction of the unit or the year. When students have a better idea of where the concepts they are learning fit into the bigger picture, they are more likely to understand how to use the processes they are learning. For example, a student may learn factoring and the quadratic formula in the same unit. However, if a student only remembers the quadratic formula and does not get to review factoring, he or she is less likely to understand the overarching picture and understand that these processes are both methods of solving quadratic equations. Therefore, a review of what has been taught over the past week or month would be useful for the student to see where factoring or using the quadratic formula comes into play, allowing him to be better at determining which process to use when given a quadratic equation with no context of which lesson or skill it may pertain to.

Reviews should be utilized to determine what students remember and what content they struggle with. Suydam (1985) makes this suggestion as a way for students to evaluate their own knowledge as well as for teachers to evaluate a student's, or an entire class', knowledge. This is especially important in determining if there are key concepts that require more practice or require more context for students to apply their skills to the next concept. When teachers give more difficult review questions, they provide a way of reviewing that promotes the retention of "vertical transfer skills," skills that will be needed when they are faced with a more difficult or novel problem (Lee, 1980).

Multiple assessments on a topic will likely result in an increase in student learning. Dempster says that "three or more tests covering the same educational objectives are likely to result in more learning if there is a progressive increase in the interval between each of the successive tests than if the interval between the tests is the same" (1991, p. 72) In other words, if a quiz is given on a topic that was recently taught (perhaps two days after the lesson), followed by a test (perhaps two weeks after the lesson) on that topic, and followed by a midterm (perhaps two months after the lesson) that also covers that topic, students are more likely to make greater strides in their learning and understanding of that one topic. In addition, Dempster also claims that "the effects of testing are greater for repeated questions than for new items" (1991, p. 72). This idea that testing older topics actually has a greater effect than testing newer topics may be a foreign one, but it aligns with the idea of spacing. Spacing is not only effective for practice (on homework or classwork), but it is also effective on tests as well.

**Challenges in Application of Cumulative Review**

In a classroom where cumulative review is not regularly applied, there are many challenges in attempting its implementation. A few of these difficulties were addressed in the applications of the research that I found.

One difficulty in implementing regular cumulative review is how students will react. Students, especially those who are much more used to blocked practice, will most likely feel that they are unable to work as quickly through a set of cumulative review problems. Carpenter (2014) states that students will often find that they make more mistakes when they are working on interleaved practice as opposed to blocked practice. This may leave students with the impression that this type of learning is more difficult (which it may be, as they are required to use more higher-order thinking skills in the process), although research shows otherwise. Students also often perceive struggling as a lack of understanding, and may give up more easily than if they “know” how to do a problem but would be unable to solve the problem outside of its given context.

Another difficulty in implementing regular cumulative review is the educational resources that are available. Most educational resources do not provide the sort of spaced or mixed practice that the research out there has shown to be useful (Carpenter, 2014). This puts extra work on the teacher to create the kind of spaced or mixed practice that will be most useful in the classroom. Although this might be more work for the teacher, it is work that will be beneficial for students and is work that is worth putting the extra effort in order to help students become more literate mathematicians.

**Conclusions**

In doing this research on cumulative review, I have a better understanding of the types of review and their benefits. I would characterize my own teaching practice as one focused on blocked practice, with some cumulative practice before any assessments. In reading about the benefits of cumulative review, mainly in the aspect of spaced practice, I plan to incorporate more spaced practice, before assessments come up. I hope to be able to incorporate even a couple of problems each day in class as a “Do Now” or “Exit Ticket” that would help students link what they are currently learning to old topics and skills that they already have. In this way, I hope to foster a type of learning where students understand mathematical content at a deeper level than they already are, and be prepared to solve any new problem that they may face later on in their mathematical education.

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